5 Arithmetic Progressions

Lesson at a Glance

1. A sequence is an arrangement of numbers according to some rule.
2. A sequence may be either a finite or an infinite.
3. A finite sequence has a finite number of terms.
4. An infinite sequence has infinite number of terms.
5. A sequence is called a progression, if its terms are either in an increasing order or a decreasing order.
6. A sequence is called an arithmetic progression if its terms are increasing or decreasing by a constant number.
7. On subtracting any term of an arithmetic progression (A.P.) from its next term, we get the common difference (d) of the A.P.
8. The common difference of an A.P. can be positive, negative or zero.
9. An A.P. can contain all the terms equal.
10. Generally, we denote first term and common difference of an A.P. by a and d respectively.
11. The general form of an A.P. is a, $a + d$, $a + 2d$,

$$a + 3d, \ldots\ldots$$

12. To solve problems, if sum of terms of an A.P. is given, then for

(i) 3 terms of an A.P. we take terms as

$$a - d, a, a + d$$

(ii) 4 terms of an A.P. we take terms as

$$a - 3d, a - d, a + d, a + 3d$$

(iii) 5 terms of an A.P. we take terms as

$$a - 2d, a - d, a, a + d, a + 2d.$$

13. nth term of an A.P. is given by $a_n = a + (n - 1)d$

This term is also called general term of the A.P.

14. If a, b, c are in A.P. then $b - a = c - b$ or $2b = a + c$.

15. If $a_1, a_2, a_3, \ldots\ldots, a_n$ is an A.P., then

$$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \ldots\ldots\ a_n - a_{n-1} = d.$$

16. If a_n is the nth term of an A.P., then its common difference is given by

$$d = a_n - a_{n-1}$$

where, a_{n-1} is the $(n - 1)$th term of the A.P.

17. In an A.P. of n terms rth term from the end $= (n - r + 1)$th term from the beginning.

18. A sequence is an A.P., if $a_n - a_{n-1}$ is independent of n.

19. A sequence is an A.P., if its nth term is a linear expression in n. In such a case the common difference is the coefficient of n.

20. The sum of first n terms of an A.P. is given by

$$S_n = \frac{n}{2}[2a + (n - 1)d].$$

21. If the last term of an A.P. is l and a is the first term, then the sum of n terms of the A.P. is given by

$$S_n = \frac{n}{2}(a + l).$$

22. If the last term, say nth term of an A.P. is l, then

$$l = a + (n - 1)d$$

23. nth term of an A.P., whose sum to n terms, S_n, is given by

$$a_n = S_n - S_{n-1}.$$

TEXTBOOK QUESTIONS SOLVED

Exercise 5.1 *(Page – 99-100)*

1. ***In which of the following situations, does the list of numbers involved make an arithmetic progression, and why?***

   (*i*) *The taxi fare after each km when the fare is ₹ 15 for the first km and ₹ 8 for each additional km.*

   (*ii*) *The amount of air present in a cylinder when a vacuum* *pump removes $\frac{1}{4}$ of the air remaining in the cylinder at a time.*

(*iii*) *The cost of digging a well after every metre of digging, when it costs ₹ 150 for the first metre and rises by ₹ 50 for each subsequent metre.*

(*iv*) *The amount of money in the account every year, when ₹ 10000 is deposited at compound interest at 8% per annum.*

Sol. (*i*) Yes. ₹ 15, ₹ 23, ₹ 31, ... is an A.P. as common difference is ₹ 8.

(*ii*) No. $1, \frac{3}{4}, \frac{9}{16}, \frac{27}{64}$, ... is not an A.P. as common difference is not constant.

(*iii*) Yes. ₹ 150, ₹ 200, ₹ 250, ... is an A.P. as common difference is ₹ 50.

(*iv*) No. ₹ $10000 \times \frac{108}{100}$, ₹ $10000 \times \left(\frac{108}{100}\right)^2$,

₹ $10000 \left(\frac{108}{100}\right)^3$,........ is not an A.P. as common difference is not constant.

2. *Write first four terms of the AP, when the first term a and the common difference d are given as follows:*

(*i*) $a = 10, \quad d = 10$ (*ii*) $a = -2, \quad d = 0$

(*iii*) $a = 4, \quad d = -3$ (*iv*) $a = -1, \quad d = \frac{1}{2}$

(*v*) $a = -1.25, \quad d = -0.25$

Sol. We will use: A.P. is $a, a + d, a + 2d$, ...

(*i*) terms are 10, 10 + 10, 10 + 20, 10 + 30, *i.e.*, 10, 20, 30, 40.

(*ii*) terms are – 2, – 2 + 0, – 2 + 0, – 2 + 0, *i.e.*, – 2, – 2, – 2, – 2.

(*iii*) terms are 4, 4 – 3, 4 – 6, 4 – 9, *i.e.*, 4, 1, – 2, – 5.

(*iv*) terms are $-1, -1 + \frac{1}{2}, -1 + 1, -1 + \frac{3}{2}$,

i.e., $-1, -\frac{1}{2}, 0, \frac{1}{2}$.

(*v*) terms are $-1.25, -1.25-0.25, -1.25-0.50, -1.25-0.75$, *i.e.*, $-1.25, -1.50, -1.75, -2.00$.

3. *For the following APs, write the first term and the common difference:*

(*i*) $3, 1, -1, -3, \ldots\ldots$ (*ii*) $-5, -1, 3, 7, \ldots\ldots.$

(*iii*) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}, \ldots\ldots$ (*iv*) $0.6, 1.7, 2.8, 3.9, \ldots\ldots.$

Sol. (*i*) $a = 3, \quad d = 1 - 3 = -2$

(*ii*) $a = -5, d = -1 + 5 = 4$

(*iii*) $a = \frac{1}{3}, \quad d = \frac{5}{3} - \frac{1}{3} = \frac{4}{3}$

(*iv*) $a = 0.6, \quad d = 1.7 - 0.6 = 1.1.$

4. *Which of the following are APs? If they form an AP, find the common difference d and write three more terms.*

(*i*) $2, 4, 8, 16, \ldots\ldots.$ (*ii*) $2, \frac{5}{2}, 3, \frac{7}{2}, \ldots\ldots.$

(*iii*) $-1.2, -3.2, -5.2, -7.2, \ldots\ldots$

(*iv*) $-10, -6, -2, 2, \ldots\ldots.$

(*v*) $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}, \ldots\ldots$

(*vi*) $0.2, 0.22, 0.222, 0.2222, \ldots\ldots.$

(*vii*) $0, -4, -8, -12, \ldots\ldots$

(*viii*) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \ldots\ldots.$ (*ix*) $1, 3, 9, 27, \ldots\ldots$

(*x*) $a, 2a, 3a, 4a, \ldots\ldots.$ (*xi*) $a, a^2, a^3, a^4, \ldots\ldots$

(*xii*) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \ldots\ldots.$

(*xiii*) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, \ldots\ldots$

(*xiv*) $1^2, 3^2, 5^2, 7^2, \ldots\ldots.$ (*xv*) $1^2, 5^2, 7^2, 73, \ldots\ldots$

Sol. (*i*) $4 - 2 = 2; 8 - 4 = 4; 16 - 8 = 8.$

Difference is not constant. Hence, it is not an A.P.

(*ii*) $\frac{5}{2} - 2 = \frac{1}{2}; 3 - \frac{5}{2} = \frac{1}{2}; \frac{7}{2} - 3 = \frac{1}{2}.$

Difference is constant, *i.e.*, $d = \frac{1}{2}$. Hence, it is an A.P.

Next three terms are:

$$\frac{7}{2}+\frac{1}{2}=4,\ 4+\frac{1}{2}=\frac{9}{2} \text{ and } \frac{9}{2}+\frac{1}{2}=5.$$

(*iii*) $-3.2 + 1.2 = -2; -5.2 + 3.2 = -2; -7.2 + 5.2 = -2.$

Difference is constant, *i.e.*, $d = -2$. Hence, an A.P.

Next three terms are:

$-7.2 - 2 = -9.2, -9.2 - 2 = -11.2$

and $-11.2 - 2 = -13.2.$

(*iv*) $-10, -6, -2, 2, \ldots\ldots\ldots$

Here, $-6 + 10 = 4, -2 + 6 = 4, 2 + 2 = 4 \ldots\ldots\ldots$

Difference is constant, *i.e.*, $d = 4$.

Hence, it is an A.P.

Three more terms are:

$2 + 4 = 6, 6 + 4 = 10$ and $10 + 4 = 14.$

(*v*) $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}, \ldots\ldots\ldots$

Here, $3+\sqrt{2}-3$

$=\sqrt{2}, 3+2\sqrt{2}-3-\sqrt{2}=\sqrt{2},$

$3+3\sqrt{2}-3-2\sqrt{2}=\sqrt{2} \ldots\ldots$

Difference is constant, *i.e.*,

$d=\sqrt{2}$. Hence, it is an A.P.

Next three terms are:

$3+3\sqrt{2}+\sqrt{2}=3+4\sqrt{2},$

$3+4\sqrt{2}+\sqrt{2}=3+5\sqrt{2}$ and

$3+5\sqrt{2}+\sqrt{2}=3+6\sqrt{2}.$

(*vi*) $0.22 - 0.2 = 0.02; 0.222 - 0.22 = 0.002.$

Difference is not constant. Hence, it is not an A.P.

(*vii*) $0, -4, -8, -12, \ldots\ldots$

Here, $-4 - 0 = -4, -8 + 4 = -4,$

$-12 + 8 = -4, \ldots\ldots$

Difference is constant, *i.e.*,

$d = -4$. Hence, it is an A.P.

Next three terms are:

$$-12-4=-16,\ -16-4=-20$$

and $-20-4=-24$.

(*viii*) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \ldots\ldots$

Here, $-\frac{1}{2}+\frac{1}{2}=0, -\frac{1}{2}+\frac{1}{2}=0, -\frac{1}{2}+\frac{1}{2}=0.$

Difference is constant, *i.e.*, $d=0$. Hence, it is an A.P.

Next three terms are:

$$-\frac{1}{2}+0=-\frac{1}{2}, -\frac{1}{2}+0=-\frac{1}{2} \text{ and } -\frac{1}{2}+0=-\frac{1}{2}.$$

(*ix*) 1, 3, 9, 27, ...

Here, $3-1=2$, $9-3=6$, $27-9=18$, ...

Difference is not constant. Hence, it is not an A.P.

(*x*) $2a-a=a$; $3a-2a=a$; $4a-3a=a$.

Difference is constant, *i.e.*, $d=a$. Hence, it is an A.P.

Next three terms are: $4a+a=5a$, $5a+a=6a$ and $6a+a=7a$.

(*xi*) $a^2-a=a(a-1)$; $a^3-a^2=a^2(a-1)$.

Difference is not constant. Hence, it is not an A.P.

(*xii*) $\sqrt{2}, 2\sqrt{2}, 3\sqrt{2}, \ldots\ldots$

$$2\sqrt{2}-\sqrt{2}=\sqrt{2};\ 3\sqrt{2}-2\sqrt{2}=\sqrt{2};$$

$$4\sqrt{2}-3\sqrt{2}=\sqrt{2}.$$

Difference is constant, *i.e.*, $d=\sqrt{2}$. Hence, it is an A.P.

Next three terms are: $4\sqrt{2}+\sqrt{2}=5\sqrt{2}=\sqrt{50}$,

$$5\sqrt{2}+\sqrt{2}=6\sqrt{2}=\sqrt{72}$$

and $6\sqrt{2}+\sqrt{2}=7\sqrt{2}=\sqrt{98}$.

(*xiii*) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, \ldots\ldots$

Here, $\sqrt{6}-\sqrt{3} \neq \sqrt{9}-\sqrt{6} \neq \sqrt{12}-\sqrt{6}$.

Difference is not constant. Hence, it is not an A.P.

(*xiv*) $1^2, 3^2, 5^2, 7^2$, ..., *i.e.*, 1, 9, 25, 49, ...

Here, $9-1=8$, $25-9=16$, $49-25=24$ ....

Difference is not constant. Hence, it is not an A.P.

(*xv*) $1^2, 5^2, 7^2$, 73 ...., *i.e.*, 1, 25, 49, 73, ...

Here, $25-1=24$, $49-25=24$, $73-49=24$, ...

Difference is constant, *i.e.*, $d=24$.
Hence, it is an A.P.

Next three terms are:

$73+24=97$, $97+24=121$, $121+24=145$.

Exercise 5.2 *(Page – 105-107)*

1. *Fill in the blanks in the following table, given that a is the first term, d the common difference and a_n the nth term of the AP:*

	a	d	n	a_n
(*i*)	7	3	8	....
(*ii*)	– 18	....	10	0
(*iii*)	....	– 3	18	– 5
(*iv*)	– 18.9	2.5	....	3.6
(*v*)	3.5	0	105	....

Sol. (*i*) $a_n = a+(n-1)d = 7+(8-1)\,3 = 7+21 = 28$.

(*ii*) $a_n = a+(n-1)d \quad \Rightarrow \quad 0 = -18+(10-1)d$

$\Rightarrow 9d = 18 \quad \Rightarrow \quad d = 2$.

(*iii*) $a_n = a+(n-1)d \quad \Rightarrow \quad -5 = a+(18-1)(-3)$

$\Rightarrow \quad a = -5+51 = 46$.

(*iv*) $3.6 = -18.9+(n-1)(2.5)$

$\Rightarrow 3.6+18.9 = (n-1)(2.5)$

$\Rightarrow \quad 22.5 = 2.5n-2.5 \quad \Rightarrow \quad 2.5n = 25$

$\Rightarrow \quad n = 10$.

(*v*) $a_n = a+(n-1)d = 3.5+(105-1)\,0 = 3.5$.

2. *Choose the correct choice in the following and justify:*

(i) *30th term of the AP: 10, 7, 4, ..., is*

(A) 97 (B) 77 (C) − 77 (D) −87

(ii) *11th term of the AP:* $-3, -\frac{1}{2}, 2, \ldots\ldots,$ *is*

(A) 28 (B) 22 (C) − 38 (D) $-48\frac{1}{2}$

Sol. (i) $a = 10, d = 7 - 10 = -3,$

$a_{30} = 10 + (30 - 1)(-3) = 10 - 87 = -77.$

Hence, option (C) is correct.

(ii) $a = -3, d = -\frac{1}{2} + 3 = \frac{5}{2};$

$a_{11} = -3 + (11 - 1)\,\frac{5}{2} = -3 + 25 = 22.$

Hence, option (B) is correct.

3. *In the following APs, find the missing terms in the boxes:*

(i) $2, \boxed{}, 26$ (ii) $\boxed{}, 13, \boxed{}, 3$

(iii) $5, \boxed{}, \boxed{}, 9\frac{1}{2}$ (iv) $-4, \boxed{}, \boxed{}, \boxed{}, \boxed{}, 6$

(v) $\boxed{}, 38, \boxed{}, \boxed{}, \boxed{}, -22$

Sol. (i) $a_1 = 2, a_3 = 26; \quad 26 = 2 + (3 - 1)\,d$

$\Rightarrow\ 24 = 2d \quad \Rightarrow\ d = 12.$

$\therefore\ a_2 = 2 + 12 = 14.$

Hence, $\boxed{14}$.

(ii) $a_2 = 13 \ \Rightarrow a + d = 13$ and $a_4 = 3 \Rightarrow a + 3d = 3$

Solving for a and d, we get $a = 18, d = -5$

$\therefore\ a_3 = 18 + 2 \times (-5) = 8.$

Hence, $\boxed{18}, \boxed{8}$.

(iii) $a_1 = 5, a_4 = \frac{19}{2}; \ \frac{19}{2} = 5 + (4 - 1)\,d$

$\Rightarrow\ 3d = \frac{9}{2} \quad \Rightarrow\ d = \frac{3}{2}$

$\therefore \quad a_2 = 5 + \frac{3}{2} = \frac{13}{2} = 6\frac{1}{2}; \; a_3 = \frac{13}{2} + \frac{3}{2} = \frac{16}{2} = 8.$

Hence, $\boxed{6\frac{1}{2}}$, $\boxed{8}$

(iv) $a = -4$ and $a_6 = 6 \Rightarrow -4 + 5d = 6$

$\Rightarrow \quad d = 2$

$\therefore \quad a_2 = -2, a_3 = 0, a_4 = 2, a_5 = 4.$

Hence, $\boxed{-2}$, $\boxed{0}$, $\boxed{2}$, $\boxed{4}$.

(v) **Hint:** $a + d = 38; \; a + 5d = -22.$

On solving, we get $a = 53$, $d = -15$. Find a_3, a_4, a_5.

4. *Which term of the AP: 3, 8, 13, 18, ..., is 78?*

Sol. Let $a_n = 78$, then $78 = 3 + (n-1)5 \Rightarrow n = 16.$

5. *Find the number of terms in each of the following APs:*

(i) *7, 13, 19, ..., 205* (ii) *18, $15\frac{1}{2}$, 13, ..., – 47*

Sol. (i) Let $a_n = 205, a = 7, d = 6$

Then, $205 = 7 + (n-1)6 \Rightarrow 198 = (n-1)6$

$\Rightarrow \quad n - 1 = 33 \Rightarrow n = 34.$

(ii) Let $a_n = -47, a = 18, d = -\frac{5}{2}.$

Then, $-47 = 18 + (n-1)\left(-\frac{5}{2}\right) \Rightarrow -65 = (n-1)\left(-\frac{5}{2}\right)$

$\Rightarrow \quad n - 1 = 26 \Rightarrow n = 27.$

6. *Check whether – 150 is a term of ~~the AP~~: 11, 8, 5, 2, ...*

Sol. Let $a_n = -150$, then $-150 = 11 + (n-1)(-3)$

$\Rightarrow \quad -161 = -3n + 3 \Rightarrow 3n = 164$

$\Rightarrow \quad n = \frac{164}{3} = 54.67 \notin \mathrm{N}.$

Hence, – 150 is not a term of the given AP.

7. *Find the 31st term of an AP whose 11th term is 38 and the 16th term is 73.*

Sol. $a + 10d = 38; \; a + 15d = 73 \Rightarrow a = -32, d = 7.$

$\therefore \quad a_{31} = -32 + 30 \times 7 = -32 + 210 = 178.$

8. *An AP consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term.*

Sol. $a_3 = 12$ and $a_{50} = 106 \Rightarrow a + 2d = 12$

and $a + 49d = 106$

Solving for a and d, we get, $a = 8, d = 2$

$\therefore \quad a_{29} = 8 + (29 - 1) \times 2 = 8 + 56 = 64.$

9. *If the 3rd and the 9th terms of an AP are 4 and – 8 respectively, which term of this AP is zero?*

Sol. $a + 2d = 4; a + 8d = -8 \quad \Rightarrow \quad a = 8, d = -2.$

Let $a_n = 0 \quad \Rightarrow \quad 8 + (n - 1)(-2) = 0 \quad \Rightarrow \quad n = 5.$

10. *The 17th term of an AP exceeds its 10th term by 7. Find the common difference.*

Sol. $a_{17} = a_{10} + 7 \quad \Rightarrow \quad a + 16d = a + 9d + 7 \quad \Rightarrow \quad d = 1.$

11. *Which term of the AP: 3, 15, 27, 39, ... will be 132 more than its 54th term?*

Sol. Let nth term of given AP is 132 more than its 54th term.

Here, $a = 3, d = 12$

$\therefore \quad a_n = a_{54} + 132$

$\therefore \quad 3 + (n - 1)12 = 3 + (54 - 1)12 + 132$

$\Rightarrow \quad (n - 1)12 = 768 \quad \Rightarrow \quad n - 1 = 64 \quad \Rightarrow \quad n = 65$

Hence, 65th term of the AP is 132 more than its 54th term.

12. *Two APs have the same common difference. The difference between their 100th terms is 100, what is the difference between their 1000th terms?*

Sol. Let common difference of two APs be d. $\{a_n\}$ and $\{a'_n\}$ are two APs:

$a_{100} - a'_{100} = 100 \quad \Rightarrow \quad (a_1 + 99d) - (a'_1 + 99d) = 100$

$\Rightarrow \quad a_1 - a'_1 = 100.$ ...(*i*)

Now, $a_{1000} - a'_{1000} = (a_1 + 999d) - (a'_1 + 999d)$

$= a_1 - a'_1 = 100...$ [From (*i*)]

13. *How many three-digit numbers are divisible by 7?*

Sol. Three-digit numbers divisible by 7 are 105, 112, ..., 994.

Let the required number of numbers be n

$\therefore \quad 994 = 105 + (n - 1)7 \Rightarrow 7n = 896 \Rightarrow n = 128.$

14. *How many multiples of 4 lie between 10 and 250?*

Sol. Multiples of 4 between 10 and 250 are 12, 16, ..., 248.

Here, $a = 12, d = 4$, let $a_n = 248$

$\therefore \quad 248 = 12 + (n - 1)4 \Rightarrow (n - 1)4 = 236$

$\Rightarrow \quad n - 1 = 59 \Rightarrow n = 1 + 59 \Rightarrow n = 60.$

15. *For what value of n, are the nth terms of two APs: 63, 65, 67, ... and 3, 10, 17, ... equal?*

Sol. $63 + (n - 1)2 = 3 + (n - 1)7$

$\Rightarrow \quad 60 = 5(n - 1) \Rightarrow n = 13.$

16. *Determine the AP whose third term is 16 and 7th term exceeds the 5th term by 12.*

Sol. $a_3 = 16$ and $a_7 = a_5 + 12$

$\Rightarrow \quad a + 2d = 16$ and $a + 6d = a + 4d + 12$

$\Rightarrow \quad d = 6$ and $a = 4.$

Hence, A.P. is 4, 10, 16, 22, ...

17. *Find the 20th term from the last term of the AP:*

3, 8, 13, ..., 253.

Sol. From the end, $a = 253, d = -5$

$\therefore$ 20th term from the end $= 253 + (20 - 1)(-5)$

$= 253 - 95 = 158.$

18. *The sum of the 4th and 8th terms of an AP is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the AP.*

Sol. $(a + 3d) + (a + 7d) = 24 \Rightarrow 2a + 10d = 24$

$\Rightarrow \quad a + 5d = 12 \qquad ...(i)$

and $(a + 5d) + (a + 9d) = 44 \Rightarrow 2a + 14d = 44$

$\Rightarrow \quad a + 7d = 22 \qquad ...(ii)$

Solving (i) and (ii), we get

$$a = -13, d = 5$$

Thus, first three terms are $-13, -8$ and -3.

19. *Subba Rao started work in 1995 at an annual salary of ₹ 5000 and received an increment of ₹ 200 each year. In which year did his income reach ₹ 7000?*

Sol. Let $a_n = 7000$. Then $7000 = 5000 + (n-1)200$

$\Rightarrow \; 2000 = (n-1)\cdot 200 \;\Rightarrow\; n = 11.$

In the year 2006, *i.e.*, 11th year, his income reaches ₹ 7000.

20. *Ramkali saved ₹ 5 in the first week of a year and then increased her weekly savings by ₹ 1.75. If in the nth week, her weekly savings become ₹ 20.75, find n.*

Sol. $a_n = 20.75$. Then $20.75 = 5 + (n-1)(1.75)$

$\Rightarrow\; 15.75 = (n-1)(1.75) \;\Rightarrow\; n - 1 = 9 \;\Rightarrow\; n = 10$

In 10th week, her savings will be ₹ 19.75.

Exercise 5.3 *(Page – 112-114)*

1. *Find the sum of the following APs:*

(*i*) *2, 7, 12, ..., to 10 terms.*

(*ii*) *– 37, – 33, – 29, ..., to 12 terms.*

(*iii*) *0.6, 1.7, 2.8, ..., to 100 terms.*

(*iv*) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}$, ..., *to 11 terms.*

Sol. (*i*) $a = 2,\ d = 5,\ n = 10.$

$$\therefore \quad S_{10} = \frac{10}{2}[4 + (10-1)5] = 5[49] = 245.$$

(*ii*) $a = -37,\ d = 4,\ n = 12$

$$\therefore \quad S_{12} = \frac{12}{2}[-74 + 11 \times 4] = 12 \times (-15) = -180.$$

(*iii*) $a = 0.6,\ d = 1.1,\ n = 100$

$$\therefore \quad S_{100} = \frac{100}{2}[1.2 + (100-1)(1.1)]$$
$$= 50 \times 110.1 = 5505.$$

(*iv*) $a = \frac{1}{15},\ d = \frac{1}{12} - \frac{1}{15} = \frac{1}{60},\ n = 11$

$$\therefore \quad S_{11} = \frac{11}{2}\left[\frac{2}{15} + (11-1)\frac{1}{60}\right] = \frac{11}{2}\left[\frac{2}{15} + \frac{1}{6}\right]$$
$$= \frac{11}{2} \times \frac{3}{10} = \frac{33}{20}.$$

2. *Find the sums given below:*

(*i*) $7 + 10\frac{1}{2} + 14 + ... + 84$

(*ii*) $34 + 32 + 30 + ... + 10$

(*iii*) $-5 + (-8) + (-11) + ... + (-230)$.

Sol. (*i*) $a = 7,\ d = \frac{21}{2} - 7 = \frac{7}{2},\ l = 84.$

$$\therefore \quad 84 = 7 + (n-1)\frac{7}{2} \quad \Rightarrow \quad 77 = (n-1)\frac{7}{2}$$

$$\Rightarrow \quad n = 23.$$

$$\therefore \quad S_{23} = \frac{23}{2}(7 + 84) = \frac{23}{2} \times 91 = \frac{2093}{2} = 1046\frac{1}{2}.$$

(*ii*) $a = 34,\ d = -2,\ a_n = 10$

$$\Rightarrow \quad 10 = 34 + (n-1)(-2)$$

$$\Rightarrow -24 = (n-1)(-2) \quad \Rightarrow \quad n = 13$$

$$\therefore \quad S_{13} = \frac{13}{2}[34 + 10] = \frac{13}{2} \times 44 = 13 \times 22 = 286.$$

(*iii*) $a = -5,\ d = -3,\ l = -230$

$$-230 = -5 + (n-1)(-3) \quad \Rightarrow \quad -225 = (n-1)(-3)$$

$$\Rightarrow \quad n = 76$$

$$\therefore \quad S_{76} = \frac{76}{2}(-5 - 230) = 38 \times (-235) = -8930.$$

3. *In an AP,*

(*i*) *given* $a = 5,\ d = 3,\ a_n = 50$, *find* n *and* S_n.

(*ii*) *given* $a = 7,\ a_{13} = 35$, *find* d *and* S_{13}.

(*iii*) *given* $a_{12} = 37,\ d = 3$, *find* a *and* S_{12}.

(*iv*) *given* $a_3 = 15,\ S_{10} = 125$, *find* d *and* a_{10}.

(*v*) *given* $d = 5,\ S_9 = 75$, *find* a *and* a_9.

(*vi*) *given* $a = 2,\ d = 8,\ S_n = 90$, *find* n *and* a_n.

(*vii*) *given* $a = 8,\ a_n = 62,\ S_n = 210$, *find* n *and* d.

(*viii*) *given* $a_n = 4,\ d = 2,\ S_n = -14$, *find* n *and* a.

(*ix*) *given* $a = 3,\ n = 8,\ S = 192$, *find* d.

(*x*) *given* $l = 28,\ S = 144$, *and there are total 9 terms. Find* a.

Sol. (*i*) $a_n = 50 \Rightarrow 5 + (n - 1)3 = 50$

$\Rightarrow (n - 1)3 = 45 \Rightarrow n - 1 = 15$

$\Rightarrow n = 16$

$$S_n = S_{16} = \frac{16}{2}(10 + 15 \times 3) = 8 \times 55 = 440.$$

(*ii*) $35 = 7 + 12d \Rightarrow d = \frac{7}{3}$

$$\therefore \quad S_{13} = \frac{13}{2}\left[14 + 12 \times \frac{7}{3}\right] = \frac{13}{2} \times 42$$

$$= 13 \times 21 = 273.$$

(*iii*) $a_{12} = 37 \Rightarrow a + 33 = 37$

$\Rightarrow a = 4$

$$S_{12} = \frac{12}{2}[8 + (12 - 1)3] = 6 \times 41 = 246.$$

(*iv*) $a + 2d = 15$ and $125 = \frac{10}{2}[2a + 9d]$

$\Rightarrow a + 2d = 15$ and $25 = 2a + 9d$

Solving, we get $a = 17, d = -1$

$\therefore a_{10} = a + 9d = 17 - 9 = 8.$

(*v*) $S_9 = 75 \Rightarrow 75 = \frac{9}{2}[2a + (9 - 1)5]$

$\Rightarrow a = -\frac{35}{3}, a_9 = -\frac{35}{3} + (9 - 1)5$

$$= -\frac{35}{3} + 40 = \frac{85}{3}.$$

(*vi*) $S_n = 90 \Rightarrow 90 = \frac{n}{2}[4 + (n - 1)8].$

$\Rightarrow 90 = \frac{n}{2}[8n - 4] \Rightarrow 90 = 4n^2 - 2n$

$\Rightarrow 2n^2 - n - 45 = 0 \Rightarrow 2n^2 - 10n + 9n - 45 = 0$

$\Rightarrow 2n(n - 5) + 9(n - 5) = 0$

$\Rightarrow (2n + 9)(n - 5) = 0$

$\Rightarrow \quad 2n + 9 = 0$ or $n - 5 = 0$

$\Rightarrow \quad n = \frac{-9}{2}$ (rejected) or $n = 5$.

$\therefore \quad a_n = a_5 = 2 + 4 \times 8 = 34$.

(*vii*) $\quad 210 = \frac{n}{2}(a + a_n) \Rightarrow 210 = \frac{n}{2}(8 + 62)$

$\Rightarrow \quad n = 6$

$a_6 = 62 \Rightarrow 62 = 8 + (6 - 1) \times d$

$\Rightarrow \quad 54 = 5d \Rightarrow d = \frac{54}{5}$.

(*viii*) $\quad a_n = 4 \Rightarrow a + (n - 1)d = 4$

$\Rightarrow \quad a + (n - 1).2 = 4 \qquad ...(i)$

$S_n = -14 \Rightarrow -14 = \frac{n}{2}[2a + (n - 1)d]$

$\Rightarrow -14 = \frac{n}{2}[8 - 4(n - 1) + 2(n - 1)]$ [Using (*i*)]

$\Rightarrow \quad -14 = \frac{n}{2}[10 - 2n] \Rightarrow -14 = n(5 - n)$

$\Rightarrow \quad n^2 - 5n - 14 = 0 \Rightarrow (n - 7)(n + 2) = 0$

$\Rightarrow \quad n - 7 = 0$ or $n + 2 = 0$

$\Rightarrow \quad n = 7, -2$ (rejected)

Substituting the value of $n = 7$ in (*i*), we get

$a + 12 = 4 \Rightarrow a = -8$.

(*ix*) $\quad S_8 = 192 \Rightarrow 192 = \frac{8}{2}[6 + (8 - 1)d]$

$\Rightarrow \quad 192 = 4[6 + 7d] \Rightarrow 48 = 6 + 7d$

$\Rightarrow \quad d = 6$.

(*x*) $\quad S_9 = \frac{9}{2}(a + l) \Rightarrow 144 = \frac{9}{2}(a + 28)$

$\Rightarrow \quad a + 28 = 32 \Rightarrow a = 4$.

4. *How many terms of the AP: 9, 17, 25, ... must be taken to give a sum of 636?*

Sol. Let $S_n = 636 \Rightarrow 636 = \frac{n}{2}[18 + (n - 1)8]$

$\Rightarrow \quad 636 = n[9 + 4n - 4] \Rightarrow 4n^2 + 5n - 636 = 0$

$\Rightarrow \quad 4n^2 + 53n - 48n - 636 = 0$

$\Rightarrow \quad n(4n + 53) - 12(4n + 53) = 0$

$\Rightarrow \quad (4n + 53)(n - 12) = 0$

$\Rightarrow \quad n = -\frac{53}{4}$ (rejected) or $n = 12$.

5. *The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.*

Sol. $a_1 = 5,\ a_n = 45,\ S_n = 400$

$\therefore \quad 400 = \frac{n}{2}[5 + 45] \quad \Rightarrow \quad n = 16$

$\therefore \quad a_{16} = 45 \quad \Rightarrow \quad 5 + 15d = 45 \quad \Rightarrow \quad d = \frac{40}{15} = \frac{8}{3}.$

6. *The first and the last terms of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?*

Sol. $a = 17,\ a_n = 350,\ d = 9$

$\therefore \quad 350 = 17 + (n - 1)9 \quad \Rightarrow \quad n = 38.$

$\therefore \quad S_{38} = \frac{38}{2}(17 + 350) = 19 \times 367 = 6973.$

7. *Find the sum of first 22 terms of an AP in which d = 7 and 22nd term is 149.*

Sol. $a_{22} = 149 \quad \Rightarrow \quad 149 = a + 21 \times 7 \quad \Rightarrow \quad a = 2$

$\therefore \quad S_{22} = \frac{22}{2}[4 + (22 - 1) \times 7] = 11 \times 151 = 1661.$

8. *Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.*

Sol. $a_2 = 14$ and $a_3 = 18 \quad \Rightarrow \quad a + d = 14$

and $\quad a + 2d = 18$

$\Rightarrow \quad a = 10,\ d = 4$

$\therefore S_{51} = \frac{51}{2}[20 + (51 - 1) \times 4] = 51 \times 110 = 5610.$

9. *If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.*

Sol. $\frac{7}{2}[2a + (7 - 1)d] = 49 \quad \Rightarrow \quad a + 3d = 7 \qquad ...(i)$

and $\frac{17}{2}[2a + (17 - 1)d] = 289 \Rightarrow a + 8d = 17$ ...(*ii*)

Solving (*i*) and (*ii*), we get

$a = 1, d = 2$

$\therefore \quad S_n = \frac{n}{2}[2 + (n - 1)2] = \frac{n}{2} \times 2n = n^2.$

10. *Show that $a_1, a_2, \ldots, a_n, \ldots$ form an AP where a_n is defined as below:*

(*i*) $a_n = 3 + 4n$ (*ii*) $a_n = 9 - 5n$

Also find the sum of the first 15 terms in each case.

Sol. (*i*) $a_n = 3 + 4n$

$d = a_n - a_{n-1} = (3 + 4n) - \{3 + 4(n - 1)\}$

$= 3 + 4n - 3 - 4n + 4 = 4$

As d is free from n, *i.e.*, d is constant. Hence $\{a_n\}$ is an A.P.

$a_1 = 7$

$\therefore \quad S_{15} = \frac{15}{2}[14 + (15 - 1)4] = \frac{15}{2} \times 70 = 525.$

(*ii*) $a_n = 9 - 5n,$

$d = (9 - 5n) - \{9 - 5(n - 1)\} = -5$

As d is free from n, *i.e.*, d is constant. Hence $\{a_n\}$ is an A.P.

Further $a_1 = 4$.

$\therefore \quad S_{15} = \frac{15}{2}[8 + 14 \times (-5)] = \frac{15}{2} \times (-62) = -465.$

11. *If the sum of the first n terms of an AP is $4n - n^2$, what is the first term (that is S_1)? What is the sum of first two terms? What is the second term? Similarly, find the 3rd, the 10th and the nth terms.*

Sol. $S_n = 4n - n^2$ ...(*i*)

$S_1 = 4 - 1 = 3 = a$

$S_2 = 8 - 4 = 4$ [From (*i*)]

$a_n = S_n - S_{n-1} = (4n - n^2) - \{4(n - 1) - (n - 1)^2\}$

$= 4n - n^2 - 4n + 4 + n^2 - 2n + 1$

$\Rightarrow a_n = 5 - 2n \Rightarrow a_2 = 1, a_3 = -1$ and $a_{10} = -15.$

12. *Find the sum of the first 40 positive integers divisible by 6.*

Sol. 6, 12, 18, ... 40 terms

$$S_{40} = \frac{40}{2}[12 + (40 - 1)6] = 20 \times 246 = 4920.$$

13. *Find the sum of the first 15 multiples of 8.*

Sol. 8 + 16 + 24 + ... 15 terms.

$$S_{15} = \frac{15}{2}[16 + (15 - 1)8] = \frac{15}{2} \times 128 = 960.$$

14. *Find the sum of the odd numbers between 0 and 50.*

Sol. Numbers between 0 and 50 are 1, 3, 5, ... 49.

Here, $n = 25$, $a = 1$, $d = 2$

$$\therefore \quad S_{25} = \frac{25}{2}[2 + (25 - 1)2] = \frac{25}{2} \times 50 = 625.$$

15. *A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: ₹ 200 for the first day, ₹ 250 for the second day, ₹ 300 for the third day, etc., the penalty for each succeeding day being ₹ 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days?*

Sol. 200 + 250 + 300 + ... 30 terms

$$S_{30} = \frac{30}{2}[400 + (30 - 1)50] = 15 \times 1850 = ₹\ 27750.$$

16. *A sum of ₹ 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is ₹ 20 less than its preceding prize, find the value of each of the prizes.*

Sol. Let prizes be a, $a - 20$, $a - 40$; $a - 60$, ... 7 times.

The sum of these prizes is ₹ 700.

$$\therefore \quad \frac{7}{2}[2a + (7 - 1)(-20)] = 700 \quad \Rightarrow \quad a - 60 = 100$$

$$\Rightarrow \quad a = 160.$$

The prizes are of ₹ 160, ₹ 140, ₹ 120, ₹ 100, ₹ 80, ₹ 60 and ₹ 40.

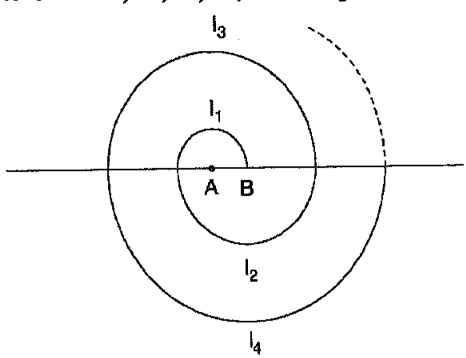
17. *In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will* *plant, will be the same as the class, in which they are studying, e.g., a section of Class I will plant 1 tree, a section of Class II will plant 2 trees and so on till Class XII. There are three sections of each class. How many trees will be planted by the students?*

Sol. Total number of trees planted by the students.

$$= 3 + 6 + 9 + \ldots\ldots + 36 = \frac{12}{2}[3 + 36] = 6 \times 39 = 234.$$

18. *A spiral is made up of successive semicircles, with centres alternately at A and B, starting with centre at A, of radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm, ... as shown in figure What is the total length of such a spiral made up of thirteen consecutive semicircles.* $\left(Take\ \pi = \frac{22}{7}\right)$

[**Hint.** *Length of successive semicircle is l_1, l_2, l_3, l_4, ... with centres at A, B, A, B, ... respectively.*]



Sol. Length of semicircle with radius 0.5 cm = 0.5 π cm

Length of semicircle with radius 1.0 cm = π cm

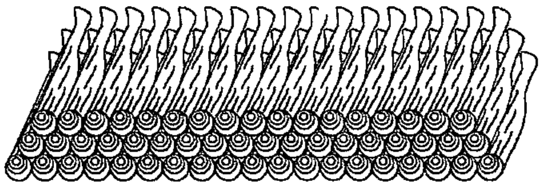
Length of semicircle with radius 1.5 cm = 1.5 π cm

These form an A.P.; which is 0.5π, π, 1.5 π, ...

$$\therefore \quad S_{13} = \frac{13}{2}[\pi + (13 - 1)0.5\pi] = \frac{13}{2} \times 7\pi$$

$$= \frac{13}{2} \times 7 \times \frac{22}{7} = 143 \text{ cm.}$$

19. *200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on (see figure). In how many rows are the 200 logs placed and how many logs are in the top row?*



Sol. $20 + 19 + 18 + \ldots\ldots.. n$ terms $= 200$

$$\frac{n}{2}[40 + (n-1)(-1)] = 200.$$

$$\Rightarrow \quad \frac{n}{2}[41 - n] = 200 \quad \Rightarrow \quad 41n - n^2 = 400$$

$$\Rightarrow \quad n^2 - 41n + 400 = 0$$

$$\Rightarrow \quad (n-25)(n-16) = 0$$

$$\Rightarrow \quad n = 25, 16$$

$$S_{25} = \frac{25}{2}[40 + 24 \times (-1)]$$

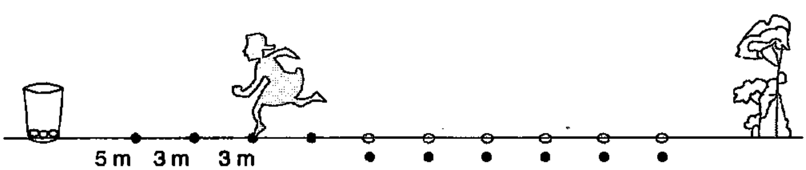
$$= \frac{25}{2} \times 16 = 200;$$

$$S_{16} = \frac{16}{2}[40 + 15 \times (-1)] = 8 \times 25 = 200$$

$n = 25$ is ruled out as $a_{25} = -4 < 0$.

$\therefore \quad n = 16$. And $a_{16} = 20 + (16-1) \cdot (-1) = 5$.

20. *In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato, and the other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line (see fig.).*



A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?

[**Hint.** *To pick up the first potato and the second potato, the total distance (in metres) run by a competitor is*
$2 \times 5 + 2 \times (5 + 3)$]

Sol. To pick up first, second, third, ... potatoes, distances covered are 10 m, 16 m, 22 m, ... These form an A.P. with $a = 10$, $d = 6$.

$$\therefore \quad S_{10} = \frac{10}{2}[20 + (10 - 1)6] = 5 \times 74 = 370 \text{ m.}$$

Exercise 5.4 *(Page – 115)*

1. *Which term of the AP: 121, 117, 113, ..., is its first negative term?*

[**Hint.** *Find n for* $a_n < 0$]

Sol. AP is 121, 117, 113, ...

Let $a_n < 0 \Rightarrow 121 + (n - 1)(-4) < 0$

$\Rightarrow 125 - 4n < 0 \Rightarrow 125 < 4n$

$\Rightarrow n > 31\frac{1}{4}$

$\Rightarrow n = 32,$

i.e., 32nd term is the first negative term.

2. *The sum of the third and the seventh terms of an AP is 6 and their product is 8. Find the sum of first sixteen terms of the AP.*

Sol. $a_3 + a_7 = 6 \Rightarrow a + 2d + a + 6d = 6$

$\Rightarrow 2a + 8d = 6 \Rightarrow a + 4d = 3$ ...(*i*)

Also $(a + 2d)(a + 6d) = 8$

$\Rightarrow (3 - 4d + 2d)(3 - 4d + 6d) = 8$ [From (*i*)]

$\Rightarrow (3 - 2d)(3 + 2d) = 8 \Rightarrow 9 - 4d^2 = 8$

$\Rightarrow 4d^2 = 1 \Rightarrow d = \pm\frac{1}{2}$

When $d = \frac{1}{2}$, $a = 1$, $S_{16} = \frac{16}{2}\left[2 + (16-1)\frac{1}{2}\right]$

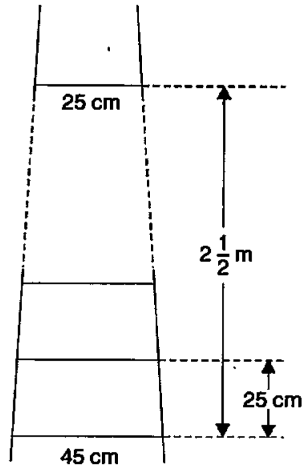
$= 8 \times \frac{19}{2} = 76$

When $d = -\frac{1}{2}$, $a = 5$, $S_{16} = \frac{16}{2}\left[10 + (16-1)\left(-\frac{1}{2}\right)\right]$

$$= 8 \times \frac{5}{2} = 20.$$

3. *A ladder has rungs 25 cm apart (see figure). The rungs decrease uniformly in length from 45 cm at the bottom to 25 cm at the top. If the top and the bottom rungs are $2\frac{1}{2}$ m apart, what is the length of the wood required for the rungs?*

$\left[\textbf{Hint:}\ \textit{Number of rungs} = \frac{250}{25} + 1\right]$



Sol. Number of rungs $= \dfrac{2\frac{1}{2}\text{ m}}{25\text{ cm}} + 1$

$$= \frac{250\text{ cm}}{25\text{ cm}} + 1 = 11$$

$$a_1 = 45 \text{ and } a_{11} = 25 \quad \Rightarrow \quad 45 + 10d = 25$$

$$\Rightarrow \qquad d = \frac{-20}{10} = -2$$

$$\therefore \qquad S_{11} = \frac{11}{2}[90 + 10 \times (-2)]$$

$$= 5.5 \times 70 \text{ cm} = 385 \text{ cm}.$$

4. *The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it. Find this value of x.*

[**Hint:** $S_{x-1} = S_{49} - S_x$]

Sol. House numbers are: 1, 2, 3, ..., x, ..., 49

According to the given conditions,

$$S_{x-1} = S_{49} - S_x$$

$$\Rightarrow \quad \frac{x-1}{2}[2 + (x-2)1] = \frac{49}{2}[2 + 48] - \frac{x}{2}[2 + (x-1)1]$$

$$\Rightarrow \qquad (x-1)(x) = 49 \times 50 - x(x+1)$$

$$\Rightarrow \quad x^2 - x = 2450 - x^2 - x \quad \Rightarrow \quad x^2 = 1225$$

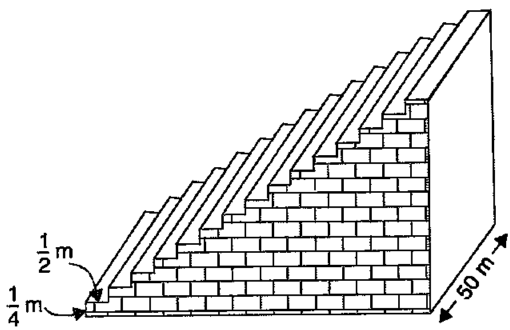
$$\Rightarrow \quad x = 35.$$

5. *A small terrace at a football ground comprises of 15 steps each of which is 50 m long and built of solid concrete.*

Each step has a rise of $\frac{1}{4}$ m and a tread of $\frac{1}{2}$ m (see figure). Calculate the total volume of concrete required to build the terrace.

[**Hint:** *Volume of concrete required to build the first step*

$$= \frac{1}{4} \times \frac{1}{2} \times 50 \ m^3]$$



Sol. Volume for the first step $= \frac{1}{4} \times \frac{1}{2} \times 50$ m^3

Volume for the second step $= \frac{2}{4} \times \frac{1}{2} \times 50$ m^3

Volume for the third step $= \frac{3}{4} \times \frac{1}{2} \times 50$ m^3

These form an A.P. which are

$\frac{1}{4} \times \frac{1}{2} \times 50, \frac{2}{4} \times \frac{1}{2} \times 50, \frac{3}{4} \times \frac{1}{2} \times 50$ ...... upto 15 terms

Here, $a = \frac{50}{8}$ and $d = \frac{100}{8} - \frac{50}{8} = \frac{50}{8}$

$$\therefore \quad S_{15} = \frac{15}{2}\left[\frac{100}{8} + 14 \times \frac{50}{8}\right] = \frac{15}{2} \times 100 = 750.$$

Hence, total volume of concrete required is 750 m^3.